Longevity Risk Transfer

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Summary

Significant increases in life expectancy is certainly a great achievement for society, although it brings one of the biggest challenges for insurance and pension markets throughout the world: managing the risk of longevity. This article presents the existing instruments for risk transfer in the international market and the principle operations occurring worldwide in order to represent how buyers of protection are adapting to risk. For the purpose of exemplifying these transactions, we discuss the calculations for Q-Forward and P-Forward contracts, that can inspire the launch products to cover deviations from the biometric assumptions, in a form regulated by CNPC Resolution 17/2015.

Key words

Longevity risk transfer. P-Forward. Q-Forward.

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Transferência de Risco de Longevidade

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Resumo

O expressivo aumento da expectativa de vida com certeza é uma grande conquista para a sociedade, no entanto, traz um dos maiores desafios existentes no mercado de seguros e previdência em todo o mundo: o gerenciamento do risco de longevidade. Este artigo apresenta os instrumentos existentes no mercado internacional para transferência do risco e as principais operações mundiais ocorridas a fim de figurar como os compradores de proteção estão se adaptando ao risco. Com o propósito de exemplificação destas transações, abordamos os cálculos para os contratos de Q-Forward e P-Forward, que podem ser inspiração para lançamento de produtos para cobertura de desvios das hipóteses biométricas, na forma do regulamentado pela Resolução CNPC 17/2015.

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Sinopsis

Transferencia de Riesgo de Longevidad

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Resumen

El expresivo aumento de la expectativa de vida seguramente es una gran conquista para la sociedad, sin embargo, trae uno de los mayores desafíos existentes en el mercado de seguros y previdencia en todo el mundo: la gestión del riesgo de longevidad. Este artículo presenta los instrumentos existentes en el mercado internacional para la transferencia de riesgo y las principales operaciones mundiales realizadas para mostrar como los compradores de protección están se adaptando al riesgo. Con el fin de ejemplificar estas transacciones, abordamos los cálculos para los contratos de Q-Forward y P-Forward, que pueden ser una inspiración para el lanzamiento de productos para la cobertura de desvíos de las hipótesis biométricas, en la forma del reglamento por la Resolución CNPC 17/2015.

Palabras-Clave


Sumario

1. Introducción. 2. Instrumentos de transferencia de riesgo. 2.1. Buy-in y buy out. 2.2 Swap de Longevidad. 2.3. Bonds. 2.4 Q-Forward y P-Forward. 2.5 Principales operaciones mundiales. 3. Mercado Brasileño. 4. Simulación de un producto de Q-Forward y P-Forward. 4.1. Estructura del producto. 4.2. Datos sobre la vida de la población a ser tratada. 4.3. Tasa de mortalidad. 4.4. Esperanza de mejoría de Mortalidad. 4.5. Premio de Riesgo. 4.6. Pacificando los contratos – Factor de Improvement (FI) 4.6.1. Contrato de Q-Forward. 4.6.1.1. Base de Mortalidad. 4.6.1.2. Improvement de Mortalidad. 4.6.1.3. Valores: premio de riesgo, valor presente del contrato NPA – Net Payoff Amount. 4.6.2 Contrato de P-Forward. 4.6.2.1 Base de mortalidad. 4.6.2.2 Improvement de Mortalidad. 4.6.2.3 Valores: premio de riesgo, valor presente del contrato y NPA – Net Payoff Amount. 5. Consideraciones Finales. 6. Referencias bibliográficas.
1. Introduction

Life expectancy in Brazil and in the wider world is gradually increasing, due to technological advances, the rise in healthcare, and the ease of access to information on disease prevention. This increase over the past decades should be recognized one of society’s more significant achievements. According to the IBGE¹, the life expectancy of Brazilians rose from 74.6 to 74.9 years from 2012 to 2013 in both sexes, and if compared to 2000, where life expectancy was 69.83 years, this represents an increase of more than five years.

This situation is viewed with some concern as it provokes changes in the demands for public policies, presenting challenges not only for the state, society and family, but also for the pension companies – closed private pensions funds, open private pensions and insurance companies – which now must deal with the challenge of how to maintain their plans and, consequently, the participants and beneficiaries protected in the long-term.

Knowing the demographic characteristics of a group of participants is extremely important for the proper management of pension plans and to maintain their solvency, especially with the occurrence of increased life expectancy. Some measures are being studied to mitigate longevity risk in the pension market, such as the increase in the minimum retirement age and transfer of pension fund risks.

In order to provide a possible solution to this type of risk, we portray, in this article, a measure that is already common practice in the European and American market, namely the transfer of longevity risk, or part of it, to an insurer, reinsurer or the financial market. We will present the main alternatives for the existing transfers in the world market, as well as a simulation of two methods that are well known worldwide, which are the P-Forward and Q-forward, having the Lee-Carter method for the calculation of its Improvement Factor based on a projection of past experiences. These products can be the inspiration for the launches of new products to cover deviations from the biometric assumptions in the form regulated by Resolution 17/2015 CNPC, which are among the possible forms of risk transfer from pension funds to insurance companies.

In Section 2, we present the options in the world market for the transfer of longevity risk. The methods that the Brazilian regulator uses to protect the parties involved in longevity risk transfer process are presented in Section 3. In Section 4, we exemplify the measurement used for the Q-forward and P-Forward products. The conclusions are presented in Section 5.

¹ IBGE – Instituto Brasileiro de Geografia e Estatística.
2. Risk Transfer Instruments

In this section, the types of longevity risk transfer transactions known internationally are addressed, be they buy-in, buy-out, swaps, bonds, Q-Forward and P-Forward. Each technique is highly dependent on its counterparty, as shown in Figure 1. Insurers are associated with buy-in, buy-out and longevity insurance, while longevity swaps are associated with investment banks and reinsurers, bonds transactions have not had success, so we will only talk about them conceptually, and the Q-Forward and P-forward processes are linked to the capital markets. The choice of each type of transaction may have a different implication for the plan. However, longevity swap, Q-Forward and P-forward can easily be adapted to the insurance concepts and be embraced by the insurance and reinsurance market, but we must bear in mind that these transactions should be viewed as life insurance, given the risks involved in the operation.

Figure 1 – Basic structure of longevity transfer in a defined benefit plan (DB)

As discussed by Neves (2008), we can see the securitization as a viable option, since the financial market has greater capacity than the insurance and reinsurance market, in addition to the financial risks tending to be independent of the longevity risk.
2.1 Buy-in and buy-out

According to the article from the *Basel Committee on Banking Supervision* (2013), a buy-in transaction occurs when an insurer assumes financial responsibility to cover the cost of benefit payments to be paid to members of a plan. The pension fund transfers a premium to the insurance company, which, in turn, ensures a periodic income stream to the pension funds, perfectly matched to insured pensions. This kind of “insurance policy” is seen as an asset by the pension plan, which guarantees the premium payments even if retirees live longer than expected. In the buy-out transactions, the assets and liabilities of a pension fund are transferred to an insurer in exchange for an up-front premium. These responsibilities with the benefits and their compensation assets are removed from the pension fund’s balance sheet and the insurer shall assume full responsibility for the payments of the beneficiaries. Figure 2 shows the structure of these products.

**Figure 2 – Structures of Buy-out and buy-in transactions**

<table>
<thead>
<tr>
<th>Buy-out</th>
<th>Buy-in</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pension Plan</strong></td>
<td><strong>Pension Plan</strong></td>
</tr>
<tr>
<td><strong>Employee</strong></td>
<td><strong>Employee</strong></td>
</tr>
<tr>
<td><strong>Assets and liabilities</strong></td>
<td><strong>Up-front premium</strong></td>
</tr>
<tr>
<td><strong>Up-front premium</strong></td>
<td><strong>Benefit payment</strong></td>
</tr>
<tr>
<td><strong>(Re)insurer</strong></td>
<td><strong>Benefit Payment</strong></td>
</tr>
</tbody>
</table>

Source: Basel Committee on Banking Supervision (2013)

Buy-in and buy-out transactions have high apparent costs, since insurers are subject to more rigorous rules that pension funds, such as the need to retain provisions that reflect the best estimate of the risk and risk-based capital. In addition, the cases of buy-out may appear too expensive, considering that any underfunding requires a fixed payment for the initial sponsor to achieve full funding before the plan is passed on to a third party.

The majority of risk transfer transactions in the UK (i.e., greater than £ 500 million) since 2007, have been buy-ins or longevity swaps. In contrast, the same operations in the United States have been buy-outs. Compared to other types of risk transfer, the buy-out has a potential disadvantage relative to the others, because the responsibility for the benefit payment to the participant and may lead to political discussions.
2.2 Longevity swaps

The theory of longevity swaps is similar to insurance made by an insurance company to a financial institution based on the current forecast of life expectancy. Thus, if life expectancy increases above a certain level, the insurance covers – or apportions, as agreed – the extra payments. In the financial market, longevity swaps are a type of derivative targeted to investors and fixed income funds interested in devices that allow the transfer of risks from pension funds and insurers to capital markets, called insurance linked securities. Neves (2008) presents the mortality swap as an alternative management and hedge of the longevity risk for insurance companies, open private pension entities and pension funds that sell annuities.

In this case of risk transfer, the plan’s sponsor makes fixed payments to the counterparty who will perform the swap, which in turn makes periodic payments that are based on the difference between actual payments and the expected benefits, therefore, the sponsor retains full responsibility for the payments of benefits to its employees.

**Figure 3 – Structure of a longevity Swap transaction**

![Diagram of a longevity swap transaction]

Source: Basel Committee on Banking Supervision (2013)

The contracting of longevity swaps is attractive due to their lower cost compared to traditional insurance and the possibility of a combination with other investment strategies. In terms of cost, for the plans that wish to transfer risk but cannot afford the cost of a traditional insurance, a longevity swap allows the gradual removal of risk at a more affordable cost. As for plans that are free from risk, longevity swaps can be combined with a benefit plan investment strategy, using interest rates and inflation swap.

Swaps and buy-in contracts are very similar, and can be used to cover the longevity risk associated with a specific subset of a given population.
One advantage of swaps is that the longevity risk can be isolated, whereas the buy-in and buy-out are transactions that usually also transfer the investment risk of the assets. Longevity swaps can also be combined with other types of derivative contracts, such as inflation, interest rate and total return swaps to create so-called “synthetic” buy-ins that transfer all the risks.

2.3 Bonds

The transfer of longevity risk by means of a bond (securities) depends on the population’s longevity experience, where the payment is related to the number of survivors in the population, so you pay more when the number of survivors is greater.

A disadvantage in relation to the swap is that the securities buyer makes a large payment to the issuer, resulting in exposure to the counterparty risk. However, the counterparty risk would be minimized if the bonds were always issued by a low risk investor that uses, for example, fixed income, and that income covers the event. The issuer may also transfer all or part of longevity risk to a reinsurer through a longevity swap contract. The structure of the product is shown in Figure 4:

Figure 4 – Structure of a bond transaction

Source: Basel Committee on Banking Supervision (2013)
So far, transactions of this kind have not occurred, although some attempts have been made, as in the case of the European Bank\(^2\) in 2004 and the World Bank\(^3\) in 2010, but these were unsuccessful due to the lack of interest of the parties. This type of transaction has a contrast with the others in that the mortality bonds, in this case, are connected to catastrophic events such as, for example, pandemics, and the others transfer the risk in about three to five years.

### 2.4 Q-Forwards and P-Forwards

These are risk transfer instruments directly linked to the capital market. In the actuarial market, the letters \(q\) and \(p\) are classified as mortality rate and survival rate, respectively. Thus, the Q-Forward is linked to the mortality risk, and P-Forward the survival risk of. It can also be seen that some texts refer to P-Forward as S-Forward, because the letter “S” more easily refers to the term survive.

These derivatives, illustrated in Figure 5, deal with the transfer of the future mortality or survival rate with a fixed rate agreed at the beginning of the contract.

**Figure 5** – Structures of Q-Forward and P-Forward operations, respectively

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2 European Bank – Bank responsible for the single European currency (Euro).
3 World Bank – international financial institution that provides loans to developing countries for capital programs.
The Q-Forward is a contract between two parties dealing with the future exchange (maturity of the contract) of an amount proportional to a fixed mortality rate that has been mutually agreed at the beginning. So, it can be likened to a zero-coupon bond, i.e. without periodic interest payment. In turn, the P-Forward uses pretty much the same calculation as the Q-Forward, but the rate used in the calculation is the survival of a given population. In figures 6 and 7, we present the details of the products.

**Figure 6 – Q-Forward timeline**

![Diagram of Q-Forward timeline](image)

The reference rate for the termination of the contract is the mortality rate, as determined by the appropriate index. In a fair market, the fixed mortality rate at which the transaction takes place defines the “future mortality rate” for the population in question.

2.5 Principle Worldwide Operations

Europe was the first market to use risk transfer operations, specifically in the United Kingdom, but from 2012, large non-UK transactions began to surface, such as for example, a $26 billion pension contract buy-out between General Motors and the Prudential, a swap of €12 billion between Aegon and Deutsche Bank and a pension buy-out of $7 billion between Verizon Communications and the Prudential.

Despite the large volumes of the transactions, they represent only a small portion of the enormous potential that exists in longevity risk transfer market.

Figure 8, shows some of the most significant transactions in the world market from the second half of 2007.
Institutions seeking to adapt to the market are increasingly inclined towards longevity transfers. For example, Scor⁴ is conducting survival reinsurance and is heavily involved with annuity, i.e. it has supported insurers in survival risk management, developing financial instruments and risk management for companies. JP Morgan⁵ has been closely involved in an attempt to establish a point of reference for the longevity market, developing standardized instruments, such as the Q-Forwards and longevity platform, LifeMetrics.

Longevity risk transfer operations are very important, given it is estimated that the total value of annuity and exposure to risks related to longevity varies from 15 trillion to 25 trillion US Dollars (CRO Forum 2010, and Biffis and Blake, 2012).

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⁴ Scor – Group of French Reinsurers with global operations. It was founded in 1970 and today is one of the five largest reinsurers in the world.
⁵ JP Morgan – Large American company, world leader in financial services. It is present in over 100 countries.
3. Brazilian Market

In Brazil, the guidelines that govern the risk transfer market are the CNSP 119/2004 and CNPC 17 / 2015 Resolutions. They establish the rules to be observed for risk transfer of pension funds to insurance companies.

CNSP Resolution 119/2004 deals with the rules to be observed by insurance companies and open private pension entities for contracting insurance and benefits plans by closed private pension entities and others. Below, we present the main articles that guide the resolution:

“Art. 1º In contracting group life insurance policies, or benefit plans for participants of plans operated by closed private pension entities, the insurance companies and open private pension entities may accept the designation of their own closed pension fund, policyholder or endorser, as the beneficiary, subject to the rules established in specific legislation.

Art. 2º In addition to that specified in art. 1 of this Resolution, the insurance companies authorized to operate in damage insurance, subject to the criteria set out in specific legislation, are permitted to market insurance in the form of loss limitation (“stop loss”) for coverage structured in the form of defined benefit, in the event of retention of part of the risk insurable by the private pension entity, in terms of franchise.

(…)”

As shown, in Brazil it is possible that closed private pension funds can be the beneficiary of a life insurance or a benefit plan for its participants and also be allowed to contract “stop loss” insurance in the case of defined benefit plans. However, we believe that this resolution should be revised so that it fits perfectly the requirements of CNPC Resolution 17/2015.

CNPC nº17 Resolution of 13 March, 2015, deals with the contracting of insurance for benefit plans operated by closed private pension entities. We also present the articles that cover the Resolution:

“Art. 1º The closed private pension entity – EFPC shall observe the provisions of this Resolution when contracting insurance that covers risks arising from benefit plans characterized as pensions.

Art. 2º The EFPC may contract specific insurance with an insurance company authorized to operate in Brazil, in order to cover the risks to the benefit plan arising from:
I – disability of participant;
II – death of participant or beneficiaries;
III – survival of the beneficiaries; and
IV – deviations from the biometric assumptions.
§ 1º The risks referred to in items of the caput may have their coverage total or partial.
§ 2º The contracting provided for in caption shall depend on the prior realization of technical studies by the EFPC6, in which it demonstrates economic, financial and actuarial viability, and approval by the Executive Board and the Advisory Board.
§ 3º The insurance contract shall be filed at the EFPC and should be made available to participants, beneficiaries, sponsors and founders when requested, being available also to the Supervisory body.

Art. 3º The forecast for contracting insurance must be included in the Regulation and its details in the actuarial technical note of the benefit plan.

Art. 4º Insurance is forbidden which envisages:
I – the payment of values directly to the participant or beneficiaries;
II - the transfers of participant and beneficiaries, subject to the provisions of § 2 of art. 33 of Supplementary Law No. 109, 2001; and
III – transfer of the guarantor reserves to the entity contracted.

Art. 5º The supervisory board may determine the contracting of insurance provided in this resolution in a form partially or fully, in order to ensure the commitments made to the participants and beneficiaries, as provided for in the benefit plan regulations."

The aforementioned resolution determents that pension funds are permitted to contract specific insurance with an insurance company authorized to operate in Brazil, in order to cover the risks of benefit plans arising from disability, death of participant of beneficiary, beneficiary survival and deviations from the biometric assumptions. We understand that all these risks are related to the life of the participant or beneficiary and, in this way, should be guaranteed by insurance companies authorized to operate life insurance.

Hence, the insurance regulator should study the possibility of reviewing CNSP Resolution 119/2004, removing the reference to “stop loss” insurance and regulating the insurance for deviations from the biometric assumptions, adapting the mortality swap risk transfer products, Q-Forward and P-Forward, described in section 2, to the Brazilian insurance market.

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6 EFPC – Closed Private Pension Entity.
4. Simulation of a Q-Forward and P-Forward product

In this section we present a method for calculating the Q-forward and P-forward products. To simulate a longevity risk transfer, we must first determine the following points concerning our product:

- Product structure – the type of product being contracted;
- Data about the life of the population to be treated – historical mortality database of the population to be contracted;
- Mortality rates – presentation of the rates used for the population to be contracted;
- Expected mortality improvements – projection of the future rate from the rates presented in the previous item; and
- Risk premium – definition of the amount to be paid by the contractor for the execution of the transaction.

The following are the items that will be used for the necessary simulations.

4.1 Product Structure

In this sub-section we highlight all the specifics of the plan being contracted, such as interest rates used until the benefits are paid, among other details.

The cases that will be presented are Q-Forward and P-Forward contracts for the US population. For the Q-Forward we assume we want to price a 10-year contract for the male population currently aged 55 to 59 years, and 55 to 64 for the P-Forward. Thus, the contract maturity ages will be 65 to 69 years and 65 to 74 respectively. For this particular example, we assume that the Q-Forward and P-Forward payment settlements are based on the average mortality rate in 2020 for each of the five individual ages for the Q-forward, and each of the ten ages for P-forward in the maturity dates.

4.2 Data about life of the population to be treated

Information about the population to be analyzed is extremely important, and the data must be quite clear and precise to avoid deviations in the calculations. We assume in our example a country that has good quality data with great experience of mortality such as the population of the United States, which allows us to build appropriate mortality rates and forecasts of improvement for this rate, and for this reason we will not be using Brazilian population data. The database was taken from the mortality.org site which began in 2000 as a collaborative project involving research teams in the Department of Demography at the University of California, in Berkeley (USA) and at the Max Planck Institute for Demographic Research (MPIDR) in Rostock (Germany).
4.3 Mortality Rate

The mortality rate used for pricing a risk transfer product is usually referred to as graduated mortality. Prior smoothing of the crude mortality rates is required at the time of the calculation in order to calculate the mortality rate that will be considered. This procedure is necessary to decrease discrepancies that may become apparent, as mortality rates for adjacent ages are very similar and highly correlated. This procedure is very important in assessing the longevity risk and in the calculation of life expectancy.

4.4 Expected mortality improvement

Improving mortality refers to relative changes in the mortality rate compared to an initial mortality table, listed as a percentage, and the pricing of longevity risk requires an estimate of the expected mortality rate, a “best estimate” of these rates at various times in the future.

If, for example, the expected mortality improvement is 1% per year, this means that every year mortality rates are 99% of the rates of the previous year, then after 10 years, the mortality rate has fallen to \((100\% - 1\%)^{10} = 90.4\%\) of the base mortality.

Some methods can be used to determine the mortality improvement to be incorporated in the pricing of the risk transfer.

- Standard mortality improvement tables: a series of tables with pre-calculated mortality improvement are available. Examples of these include the Scale-AA in the USA (Society of Actuaries (2000)) and the tables published by the Continuous Mortality Investigation in United Kingdom (CMI (2007) and CMI (2009));

- Mortality projection based on past experiences: this involves the projection of future mortality improvements using data from the actual historical experience of the lives in question; and

- mortality projection based on past experiences of a different population: this involves a projection of future mortality improvements from historical data experience of a different life group, such as the national population, or the insured population.

Independent from the population on which the projections are being based, projections based on past experiences can be performed in various ways, for example:

- historical mortality improvements: if sufficient historical data is available, we can calculate the mortality improvements that have taken place in the past. These historical improvements can then be used as an estimate of future expected improvement. This approach has the advantage of simplicity, but may not be effective for large groups; and
Mortality projection models: a large number of projection models are available, and generate future mortality rates from historical mortality and population data. These models are more complex than the projections based directly on historical improvements. They are, however, more realistic in the way they deal with the effects of the group, for example. Among these models are included: Lee and Carter (1992), Renshaw and Haberman (2006), Debonneuil (2010), the so-called p-spline models (Durban et al, 2002), the series of Cairns-Blake-Dowd models (Cairns et al. (2006) and (2007)) and articles using SUTSE structural models from Neves et al. (2015, 2016). In this article, for the mortality projection calculation presented below, we use the Lee and Carter method (1992).

4.5 Risk premium

The risk premium is the final piece of the puzzle and is the cost that a hedge provider will charge to assume the longevity risk from a purchaser of protection wishing to reduce or eliminate the risk. The risk premium can be incorporated in the form of different prices. For example, through adjustments to the basic mortality rates, adjustments in the rates of improvement, advance payments, or a combination of all these. There are a number of factors that determine the size of the risk premium as follows:

- Factors related to investors:
  - investor demand;
  - availability of return in other products (opportunity cost);
  - benefit of diversification;
  - the maturity of the instrument that is transferring the risk; and
  - ability to provide liquidity.

- Hedge provider/intermediary/(re)insurer:
  - capital application;
  - risk appetite;
  - return on equity; and
  - diversification of “positions”.

- Other factors:
  - size of the population to be insured;
  - quality and quantity of data available;
  - size of responsibility;
  - type of instrument that will transfer the risk; and
  - amount of credit risk in the transaction.
4.6 Contract Pricing – Improvement Factor (IF)

After predicting the future mortality using the Lee-Carter method using R software\(^7\), we calculate the improvement factor (IF) of the central mortality rate for each year and age. In Table 1, we show the projections of the central mortality rate for ages 65 to 69 and projected to the year \(t + 11\).

### Table 1 – Forecast for central mortality rates

<table>
<thead>
<tr>
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<th>(t=1)</th>
<th>(t=2)</th>
<th>(t=3)</th>
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<tr>
<td>65</td>
<td>0.01550727</td>
<td>0.01531355</td>
<td>0.01512226</td>
<td>0.01493335</td>
<td>0.01474858</td>
<td>0.01456259</td>
<td>0.01438058</td>
<td>0.01420203</td>
<td>0.01402364</td>
<td>0.01384846</td>
<td>0.01367546</td>
</tr>
<tr>
<td>66</td>
<td>0.01669353</td>
<td>0.01648940</td>
<td>0.01628599</td>
<td>0.01608059</td>
<td>0.01588667</td>
<td>0.01569069</td>
<td>0.01549713</td>
<td>0.01530596</td>
<td>0.01511571</td>
<td>0.01493067</td>
<td>0.01474469</td>
</tr>
<tr>
<td>67</td>
<td>0.01812585</td>
<td>0.01793029</td>
<td>0.01773836</td>
<td>0.01754532</td>
<td>0.01735238</td>
<td>0.01716042</td>
<td>0.01696879</td>
<td>0.01677798</td>
<td>0.01658830</td>
<td>0.01640193</td>
<td>0.01621838</td>
</tr>
<tr>
<td>69</td>
<td>0.02170879</td>
<td>0.02145439</td>
<td>0.02120288</td>
<td>0.02099447</td>
<td>0.02078073</td>
<td>0.02056860</td>
<td>0.02035621</td>
<td>0.02014327</td>
<td>0.01993035</td>
<td>0.01971944</td>
<td>0.01950944</td>
</tr>
</tbody>
</table>

Considering that the central mortality rate \((m_x,t)\) and the probability of death \(q_x,t\) are directly related, through the mathematical function below:

\[
q_x = \frac{m_x}{1 + \frac{m_x}{2}}
\]

With this, we obtain, in Table 2, the projected probabilities of death which are used in this article:

### Table 2 – Forecast for the probability of death

<table>
<thead>
<tr>
<th>(q_x)</th>
<th>(t=1)</th>
<th>(t=2)</th>
<th>(t=3)</th>
<th>(t=4)</th>
<th>(t=5)</th>
<th>(t=6)</th>
<th>(t=7)</th>
<th>(t=8)</th>
<th>(t=9)</th>
<th>(t=10)</th>
<th>(t=11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>0.01538796</td>
<td>0.01519719</td>
<td>0.01500875</td>
<td>0.01482677</td>
<td>0.01464388</td>
<td>0.01445732</td>
<td>0.01429161</td>
<td>0.01412094</td>
<td>0.01399199</td>
<td>0.01387325</td>
<td>0.01375588</td>
</tr>
<tr>
<td>66</td>
<td>0.01655714</td>
<td>0.01636546</td>
<td>0.01617486</td>
<td>0.01599168</td>
<td>0.01580513</td>
<td>0.01561577</td>
<td>0.01542443</td>
<td>0.01523031</td>
<td>0.01503537</td>
<td>0.01484043</td>
<td>0.01464553</td>
</tr>
<tr>
<td>67</td>
<td>0.01796056</td>
<td>0.01777444</td>
<td>0.01758247</td>
<td>0.01739110</td>
<td>0.01720054</td>
<td>0.01690869</td>
<td>0.01661124</td>
<td>0.01630520</td>
<td>0.01600008</td>
<td>0.01579514</td>
<td>0.01558914</td>
</tr>
<tr>
<td>69</td>
<td>0.02147568</td>
<td>0.02122664</td>
<td>0.02098042</td>
<td>0.02073304</td>
<td>0.02048639</td>
<td>0.02023969</td>
<td>0.01999264</td>
<td>0.01974534</td>
<td>0.01949790</td>
<td>0.01924936</td>
<td>0.01899968</td>
</tr>
</tbody>
</table>

With the corresponding values for \(q_x\) found, we can generate the gain for each age from the formula:

\[
Gain = \frac{q_{xt} - q_{xt+1}}{q_{xt}}
\]

Table 3, shows the projected longevity gains (improvement):

### Table 3 – Projected longevity gains for age and time.

<table>
<thead>
<tr>
<th>(Gain)</th>
<th>(t=1)</th>
<th>(t=2)</th>
<th>(t=3)</th>
<th>(t=4)</th>
<th>(t=5)</th>
<th>(t=6)</th>
<th>(t=7)</th>
<th>(t=8)</th>
<th>(t=9)</th>
<th>(t=10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>1,2369664%</td>
<td>1,2398140%</td>
<td>1,2399303%</td>
<td>1,2400533%</td>
<td>1,2401585%</td>
<td>1,2402705%</td>
<td>1,2403811%</td>
<td>1,2404903%</td>
<td>1,240583%</td>
<td>1,2407048%</td>
</tr>
<tr>
<td>66</td>
<td>1,2234963%</td>
<td>1,2236198%</td>
<td>1,2237417%</td>
<td>1,2238703%</td>
<td>1,2239811%</td>
<td>1,2240987%</td>
<td>1,2242148%</td>
<td>1,2243295%</td>
<td>1,2244428%</td>
<td>1,2245548%</td>
</tr>
<tr>
<td>67</td>
<td>1,2169956%</td>
<td>1,2171282%</td>
<td>1,2172592%</td>
<td>1,2173967%</td>
<td>1,2175165%</td>
<td>1,2176428%</td>
<td>1,2177566%</td>
<td>1,2178909%</td>
<td>1,2181012%</td>
<td>1,2182194%</td>
</tr>
<tr>
<td>68</td>
<td>1,1828926%</td>
<td>1,1830306%</td>
<td>1,1831669%</td>
<td>1,1833095%</td>
<td>1,1834348%</td>
<td>1,1835664%</td>
<td>1,1836965%</td>
<td>1,1838250%</td>
<td>1,1839520%</td>
<td>1,1840775%</td>
</tr>
<tr>
<td>69</td>
<td>1,1596370%</td>
<td>1,1597813%</td>
<td>1,1599239%</td>
<td>1,1600726%</td>
<td>1,1602042%</td>
<td>1,1603420%</td>
<td>1,1604782%</td>
<td>1,1606128%</td>
<td>1,1607459%</td>
<td>1,1608774%</td>
</tr>
</tbody>
</table>

\(^7\) R – Free software, which runs on a wide variety of platforms. It uses practical language that provides a variety of statistical and graphical techniques.
Having made the appropriate calculations from the above tables, we found the improvement factor from the average of gains \( t + 1 \) to \( t + 10 \) for ages 65 to 69, generating \( IF = 1.205\% \).

**4.6.1 Q-Forward Contract**

Table 4, shown below, contains mortality rates for the population for the year 2010, which reflects the beginning of the transaction. The mean mortality rate for this population that year is simply the average of the mortality rates for each age.

**Table 4 – Mortality basis at the start of the transaction**

<table>
<thead>
<tr>
<th>Age</th>
<th>q(2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>1.59%</td>
</tr>
<tr>
<td>66</td>
<td>1.71%</td>
</tr>
<tr>
<td>67</td>
<td>1.89%</td>
</tr>
<tr>
<td>68</td>
<td>2.00%</td>
</tr>
<tr>
<td>69</td>
<td>2.19%</td>
</tr>
<tr>
<td>Mean</td>
<td>1.88%</td>
</tr>
</tbody>
</table>

**4.6.1.1 Mortality Basis**

We assume that the best estimate of the mortality improvement factor for the male population 65 to 69 years of age is 1.205\% per year according to section 4.6. This means that the best estimate of the mortality rate in any year is 98.795\% of the previous year. In addition, the best estimate of the mortality rate at the contract maturity, 2020 in our example, is 88.58\% of the mortality rate at the beginning of the contract \((1 – 1.205\%)^{10} = 88.58\%\).

These mortality improvements are incorporated in the mortality table shown in Table 5. The best estimated aggregate mortality rate in 2020 for this group is the average of the best estimates of the individual mortality rates. The average of these rates produces the best estimated aggregate as follows:

- \( Q_{BE}(2010:2020) = \text{Best estimate for aggregate mortality rate in 2020 in this transaction.} \)

- \( Q_{BE}(2010:2020) = 1.878\% \times (1-1.205\%)^{10} = 1,664\% \)

The notation \( Q_{BE}(2010:2020) \) signifies the best estimate in 2020 based on 2010.
4.6.1.3 Values:

risk premium, present value of the contract and NPA – Net Pay off Amount

We assume in this example, a rate of 1% for the risk premium. So, to have a risk premium account, we must consider the cash flow based on the mortality rate of 2.205% per year, since the best mortality estimate per year is 1.205%.

- \( q_{\text{forward}} = \text{Fixed rate paid in 2020 as determined at the start of the transaction} \)
- \( q_{\text{forward}} = q_{\text{realized}} * (100\% – \text{Improvement} – \text{Risk premium})^n = 1.503\% \)
- \( q_{\text{forward}} = 1.878\% * (1- 1.205\% – 1\%)^{10} = 1.503\% \),

where \( q_{\text{realized}} \) is the probability of death observed in the period (table 4).

Note that the forward rate differs from the best estimate rate because of the risk premium. The risk premium paid by the hedger of the operation in monetary terms is given by the present value of the notional amount multiplied by the difference in rates. Assuming a discount rate of 5% and the amount of R$ 100,000,000.00 as the value of this Q Forward contract, we find the present value of the risk premium below:

- \( \text{PV of the risk premium} = \text{R} 100,000,000.00 * (1.664\% – 1.503\%) / (1+5\%)^{10} \)
- \( \text{PV of the risk premium} = \text{R} 98,840.03 \)

The present value of the contract is obtained as follows:

\[ \text{PV of Contract} = \text{BRL } 100,000,000.00 * 1.503\% / (1+5\%)^{10} = \text{BRL } 922,711.62 \]

The only cash flow exchanged in a Q-Forward transaction happens on maturity, in the liquidation of the contract where compensation is allowed, so that only one of the counterparties should pay, calculated by the NPA (Net Payoff Amount) as below:

\[ \text{NPA}(T) = \text{Notional} \times [Q_{\text{realized}}(0:T) – Q_{\text{forward}}(0:T)] \]
Assuming a $Q_{\text{realized}}$ at the maturity of the contract, i.e., 2020 de 1.40%, we have:

\[
\text{NPA (2020)} = 100,000,000.00 \times (1.40\% - 1.503\%) = -\text{BRL 103,000.00}
\]

Assuming a $Q_{\text{realized}}$ at the maturity of the contract, i.e. 2020 de 1.60%, we have:

\[
\text{NPA (2020)} = 100,000,000.00 \times (1.60\% - 1.503\%) = \text{BRL 97,000.00}
\]

If the value of the NPA is positive, the cash flow will be a value received by the contractor. If negative, this cash flow will be a value to be paid by the contractor.

### 4.6.2 P-Forward Contract

Table 6, below, is the mortality rate for the study population aged between 65 and 74 years in 2010, as stated in item 4.2 of this text.

#### 4.6.2.1 Mortality Basis

**Table 6 – Mortality Basis at the start of the transaction**

<table>
<thead>
<tr>
<th>Age/Year</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>1.59%</td>
</tr>
<tr>
<td>66</td>
<td>1.71%</td>
</tr>
<tr>
<td>67</td>
<td>1.89%</td>
</tr>
<tr>
<td>68</td>
<td>2.00%</td>
</tr>
<tr>
<td>69</td>
<td>2.19%</td>
</tr>
<tr>
<td>70</td>
<td>2.38%</td>
</tr>
<tr>
<td>71</td>
<td>2.55%</td>
</tr>
<tr>
<td>72</td>
<td>2.84%</td>
</tr>
<tr>
<td>73</td>
<td>3.05%</td>
</tr>
<tr>
<td>74</td>
<td>3.38%</td>
</tr>
</tbody>
</table>
4.6.2.2 Mortality Improvement

Below, in Table 7, we calculate the best estimate of survival for a contract with maturity in 10 years. First, we apply the improvement factor year on year from 2010 to the mortality basis extracted earlier, in item 4.6.2.1.

Table 7 – Estimation of probabilities of mortality using an improvement of 1.205% per annum applied to mortality basis of 2010

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>1.59%</td>
<td>1.57%</td>
<td>1.55%</td>
<td>1.53%</td>
<td>1.51%</td>
<td>1.50%</td>
<td>1.48%</td>
<td>1.46%</td>
<td>1.44%</td>
<td>1.43%</td>
<td>1.41%</td>
</tr>
<tr>
<td>66</td>
<td>1.71%</td>
<td>1.69%</td>
<td>1.67%</td>
<td>1.65%</td>
<td>1.63%</td>
<td>1.61%</td>
<td>1.59%</td>
<td>1.57%</td>
<td>1.55%</td>
<td>1.53%</td>
<td>1.51%</td>
</tr>
<tr>
<td>67</td>
<td>1.89%</td>
<td>1.87%</td>
<td>1.84%</td>
<td>1.82%</td>
<td>1.80%</td>
<td>1.78%</td>
<td>1.76%</td>
<td>1.74%</td>
<td>1.72%</td>
<td>1.69%</td>
<td>1.67%</td>
</tr>
<tr>
<td>68</td>
<td>2.00%</td>
<td>1.98%</td>
<td>1.95%</td>
<td>1.93%</td>
<td>1.91%</td>
<td>1.88%</td>
<td>1.86%</td>
<td>1.84%</td>
<td>1.82%</td>
<td>1.79%</td>
<td>1.77%</td>
</tr>
<tr>
<td>69</td>
<td>2.19%</td>
<td>2.16%</td>
<td>2.14%</td>
<td>2.11%</td>
<td>2.09%</td>
<td>2.06%</td>
<td>2.04%</td>
<td>2.01%</td>
<td>1.99%</td>
<td>1.96%</td>
<td>1.94%</td>
</tr>
<tr>
<td>70</td>
<td>2.38%</td>
<td>2.35%</td>
<td>2.32%</td>
<td>2.29%</td>
<td>2.27%</td>
<td>2.24%</td>
<td>2.21%</td>
<td>2.19%</td>
<td>2.16%</td>
<td>2.13%</td>
<td>2.11%</td>
</tr>
<tr>
<td>71</td>
<td>2.55%</td>
<td>2.52%</td>
<td>2.49%</td>
<td>2.46%</td>
<td>2.43%</td>
<td>2.40%</td>
<td>2.37%</td>
<td>2.34%</td>
<td>2.31%</td>
<td>2.29%</td>
<td>2.26%</td>
</tr>
<tr>
<td>72</td>
<td>2.84%</td>
<td>2.81%</td>
<td>2.77%</td>
<td>2.74%</td>
<td>2.71%</td>
<td>2.67%</td>
<td>2.64%</td>
<td>2.61%</td>
<td>2.58%</td>
<td>2.55%</td>
<td>2.52%</td>
</tr>
<tr>
<td>73</td>
<td>3.05%</td>
<td>3.01%</td>
<td>2.98%</td>
<td>2.94%</td>
<td>2.91%</td>
<td>2.87%</td>
<td>2.84%</td>
<td>2.80%</td>
<td>2.77%</td>
<td>2.73%</td>
<td>2.70%</td>
</tr>
<tr>
<td>74</td>
<td>3.38%</td>
<td>3.34%</td>
<td>3.30%</td>
<td>3.26%</td>
<td>3.22%</td>
<td>3.18%</td>
<td>3.14%</td>
<td>3.10%</td>
<td>3.07%</td>
<td>3.03%</td>
<td>2.99%</td>
</tr>
</tbody>
</table>

In Table 8, from the probabilities of death, we calculate the year on year survival probabilities of a person aged 65. We obtain the probability of a person aged 65 years at the beginning of the contract making it alive to the end of the contract, i.e. in 2020, this probability is 80.14%.

Table 8 – Estimate of the survival probability of a person aged 65 years

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>98.43%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>96.79%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>67</td>
<td></td>
<td>95.02%</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>68</td>
<td></td>
<td></td>
<td>93.21%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>69</td>
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<td></td>
<td></td>
<td>91.29%</td>
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<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>89.27%</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>87.18%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>72</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>84.93%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>73</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>82.61%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>80.14%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.6.2.3 Values:

The risk premium, in terms of an increase in mortality improvement, is assumed at 1% per annum on the best estimate. In other words, if the best estimated mortality improvement rate is 1.205%, then to have a risk premium account, we need to calculate a survival rate based on a mortality improvement of 2.205% per year. We arrive at a forward rate of 81.21%. This forward rate differs from the best estimated mortality rate due to the risk premium. So, the risk premium corresponds to an increase of 1.07% compared to the best estimate of the survival rate at the end of 2020.

The price of the risk premium paid by hedger in monetary terms is given by the present value of the notional amount multiplied by the difference between the best estimate of the survival rate and the forward survival rate (P-Forward), as shown below:

\[
PV \text{ of risk premium} = \text{BRL} \ 100.000.000.00 \times (81.21\% - 80.14\%) / (1 + 5\%)^{10}
\]

\[
PV \text{ of risk premium} = \text{BRL} \ 656,887.18
\]

The price for this P-Forward contract is simply the present value of the two legs of the contract: the “fixed leg” and the “floating leg”. The price of the fixed leg is the forward rate – which is given from the present value of the notional amount multiplied by the forward survival rate. At the start of the transaction, if we ignore any offer of spread, the assumed rate of discount will be 5%.

\[
PV \text{ of the contract} = \text{BRL} \ 100.000.000.00 \times 81.21\% / (1 + 5\%)^{10} = \text{BRL} \ 49,855,895.32
\]

As explained in item 4.6.1.3, the NPA is the NET value of the transaction to occur only in a single cash flow, at the maturity of the contract. According to demonstration below:

\[
\text{NPA}(T) = \text{Notional} \times [\bar{Q}_{\text{realized}}(0:T) - \bar{Q}_{\text{Forward}}(0:T)]
\]

- Assuming a \(\bar{P}_{\text{realized}}\) at the maturity of the contract, i.e., 2020 of 80%, we have:

\[
\text{NPA}(2020) = 100,000,000.00 \times (80\% - 81.21\%) = - \text{BRL} \ 1,210,000.00 \rightarrow \text{Value payable by the contractor.}
\]

- Assuming a \(\bar{P}_{\text{realized}}\) at maturity of the contract, i.e., 2020 de 82%, we have:

\[
\text{NPA}(2020) = 100,000,000.00 \times (82\% - 81.21\%) = \text{BRL} \ 790,000.00 \rightarrow \text{Value receivable by the contractor.}
\]
5. Final Considerations

We understand that there are many specifics for longevity risk in view of the lack of one ideal management. As a recent move, the insurance and pension fund markets are adopting new regulations which not only allow a more accurate assessment of risk but also enforce more effective rules of management. In the global market, companies are already starting to focus on this market through existing instruments, as discussed in this work.

The variable mortality is the main demographic assumption of a pension plan, directly affecting the actuarial and financial aspects of those products. Consequently, the measurement and management of risks from these rates, both for mortality and survival, becomes essential to avoid damage to participants and beneficiaries.

Given the publication of Resolution CNPC 17/2015, which allows the transfer of risk from pension funds to insurance companies, we have presented in this article, the international products of longevity risk transfer that are waiting to inspire the Brazilian pensions market and its regulatory bodies and supervisors to create attractive products covering risks safely and efficiently.

The Q-Forward and P-forward products can be adapted to transfer part of pension fund risks to insurance companies authorized to operate in life. For example, the risk premium, presented in Section 4, could be a proxy for the insurance premium. In turn, the final determination of the contract value, as shown in section 4, must take into account the amount paid for insurance, if done at the beginning of the contract, and the prevailing rules for insurance operation.

6. Bibliographical References


