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A Different View on Risk Capital for Capitalized Insurance Products and its Respective Proposed Modelling

Lucas Barbosa Figueiredo Vilas Boas
Lucas holds a bachelor's degree in Actuarial Science from the University of São Paulo and a master's degree from City University of London. Having worked on the actuarial consulting firms Willis Towers Watson and PricewaterhouseCoopers, is now a part of the actuarial team of the joint venture Zurich-Santander. His professional experience was focused on the last 4 years in actuarial evaluation for insurance and reinsurance companies, but has participated in a variety of engagements such as pension funds evaluation, benefit and methods harmonization and loyalty and rewards programs. His main professional interests are: stochastic methods for actuarial evaluation, solvency and minimal capital requirements and the development of the actuarial function.
lucas.boas@zurich-santander.com.br

Summary

The use of the risk capital calculation as a tool to risk management departments is becoming increasingly more important to insurance companies across the world. Not only for the purpose to disclose to the external user information about the company's risk appetite, but as an analytical tool so the company can make decisions more well-structured regarding their own perception of the risk that the company faces.

When managers make business decisions, such as sale incentives, based not only on how profitable a product is but based also on the risks that such product generates – for instance is it better to receive premium as a lump sum at the beginning of the contract to avoid credit risk or is it better to have a stable inflow of premium? –, an area focused on delivering the best results for risk capital is not only a compliance issue, but also a business partner in strategical decisions.

In this context, this paper intends to bring this reality to the Brazilian culture, where the super high interest and premium rates era is coming to an end and the operational profit has more importance than ever. With this shift in the culture, a better understanding regarding the risks faced by the company is mandatory.

This Project builds a model to calculate the minimal capital requirement for the underwriting risks of capitalized operations based on nested simulations of the main parameters in the actuarial liability calculation: the mortality pattern and the interest rate.

Also, this paper delivers a stress analysis of the assumptions and presents an interesting comparability with the standard formula developed by the Brazilian insurance regulator.

Key Words

Risk capital, capital management, actuarial liability, stochastic actuarial liability.

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Sinopse

Uma Perspectiva Diferente sobre o Capital de Risco de Subscrição para Produtos Capitalizados e uma Modelagem Proposta

Lucas Barbosa Figueiredo Vilas Boas
Lucas é atuário formado pela Universidade de São Paulo e mestre em ciências atuariais pela *City University of London*. Tendo trabalhado nas consultorias Willis Towers Watson e PricewaterhouseCoopers, atualmente faz parte da equipe atuarial da *joint venture* Zurich-Santander. Sua experiência profissional nos últimos 4 anos foi focada em avaliação e auditoria atuarial de reservas para seguradoras e resseguradoras, mas já participou de projetos relacionados à avaliações de fundos de pensão, harmonização de benefícios e programas de recompensa. Seus principais interesses são: métodos estocásticos para avaliações atuariais, solvência e capital mínimo requerido e desenvolvimento da função atuarial.
lucas.boas@zurich-santander.com.br

Resumo

A utilização do cálculo de capital de risco como ferramenta de gestão de riscos vem se tornando cada vez mais importante para as companhias seguradoras no cenário mundial, não apenas uma necessidade na divulgação das demonstrações financeiras e demais demandas de usuários externos, mas também como uma ferramenta que auxilia a companhia a tomar decisões mais bem estruturadas em relação ao risco que suas próprias operações enfrentam.

Quando gestores passam a tomar decisões de negócio, como incentivo de vendas, não apenas com base na lucratividade de determinados produtos, como também nos riscos que eles geram – como, por exemplo, preferência por produtos com pagamento de prêmio único em vez de produtos com pagamento de prêmio mensal, para não gerar um risco de crédito com o ativo a receber, ainda que em detrimento de um fluxo de caixa longo –, uma área dentro das companhias focada no desenvolvimento e evolução de modelos para gerir estes riscos se torna não apenas necessária para fins regulatórios, mas também parceira nas decisões estratégicas.

Neste contexto, este trabalho tenta trazer esta realidade para o cenário brasileiro, onde tempos de resultados financeiros ótimos e ajustes de precificação fizeram com que resultados operacionais não tivessem a mesma importância que o resultado reportado. Com a mudança neste paradigma, uma melhor compreensão dos riscos assumidos pelas companhias é mandatória.

O texto propõe um modelo para o cálculo do capital mínimo requerido pelo risco de subscrição de operações, estruturadas em regime de capitalização, com base em simulações aninhadas das principais premissas em um cálculo de provisões matemáticas: padrão de mortalidade e taxa de juros.

O artigo traz, ainda, análise de *stress* nas premissas do modelo proposto e apresenta uma comparabilidade interessante com a fórmula padrão desenvolvida pelo órgão regulador do mercado segurador brasileiro.

Palavras-Chave

Capital de risco, gestão de capital, passivo atuarial, passivos atuariais estocásticos.

Sumário

Sinopsis

Una perspectiva diferente sobre el capital de riesgo de suscripción de productos capitalizados y una propuesta de modelo.

Lucas Barbosa Figueiredo Vilas Boas
Lucas es un actuario con un título de la Universidad de São Paulo y maestro en ciencias actuariales en la City, University de Londres. Después de haber trabajado con los consultores de Willis Towers Watson y PricewaterhouseCoopers, ahora forma parte del equipo actuarial de afiliados Zurich-Santander. Su experiencia profesional en los últimos 4 años se ha centrado en la evaluación y auditoría de las reservas actuariales para aseguradores y reaseguradores, pero ha participado en proyectos relacionados a revisiones de fondos de pensiones, armonización de beneficios y programas de recompensa. Sus intereses principales son: métodos estocásticos para las evaluaciones actuariales, solvencia y mínimo capital requerido, así como el desarrollo de la función actuarial.
Lucas.Boas@Zurich-Santander.com.br

Resumen

El uso de cálculo de capital de riesgo como una herramienta para la gestión de riesgos se ha vuelto cada vez más importante para las compañías de seguros en el escenario mundial, no sólo una necesidad en la divulgación de estados financieros y otras demandas de usuario externo, sino también en una herramienta que ayuda a la empresa a tomar mejores decisiones, basado en una estructura relacionada al riesgo en sus propias operaciones.

Cuando los gerentes toman decisiones de negocios, como incentivos de ventas, no sólo basados en la rentabilidad de determinados productos, sino también en los riesgos que generan – como, por ejemplo, la preferencia por productos con pago de una prima única en lugar de productos con pago de una prima mensual, para así, no generar un riesgo crediticio con el activo a recibir, aunque a costa de un extenso flujo de caja –, una área dentro de las empresas con enfoque en el desarrollo y evolución de los modelos para la gestión de este tipo de riesgos, es necesaria, no solo para fines regulatorios, sino también como un socio en la toma de decisiones estratégicas.

En este contexto, este trabajo pretende llevar este hecho al escenario brasileño, donde tiempos de excelentes resultados financieros y ajustes de precios, hicieron que los resultados operacionales no tuvieran la misma importancia que el resultado reportado. Con los cambios en este paradigma, la comprensión de los riesgos que toman las compañías se vuelve mandatorio.

El texto propone un modelo para el cálculo del capital mínimo requerido por el riesgo de suscripción operacional, estructuradas bajo un régimen de capitalización, basado en las simulaciones estructuradas derivadas de las principales primicias en un cálculo de las provisiones matemáticas: estándar de mortalidad y tasa de intereses.

El articulo trae, aún, el análisis de stress en las primicias del modelo propuesto y presenta una comparación interesante con la fórmula estándar desarrollada por el organismo regulador del mercado de seguros brasileño.

Palabras-Clave

Venture capital, gestión de capital, pasivos actuariales, pasivos

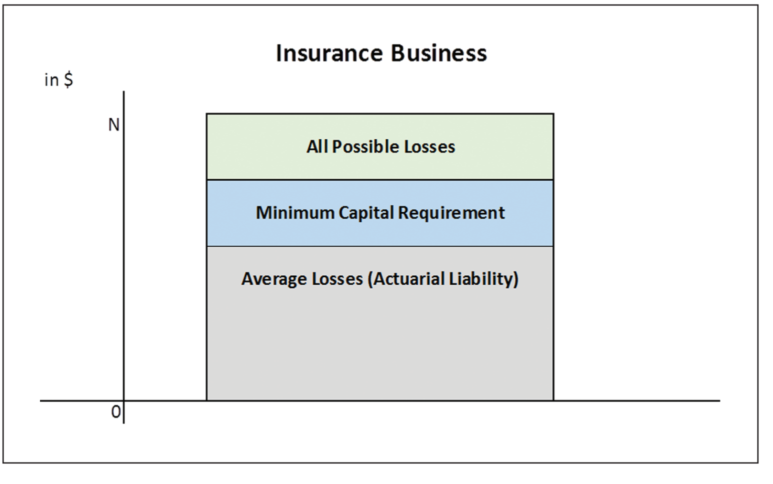
Sumario

1. Introduction

A well-known tool to Risk Management departments across virtually all industries, from construction to retail, to quantify the variability of losses from certain risks exposures is known as the Risk Capital Management. For the financial services industry, the Risk Capital Management went from a magnificent tool – from a statistical perspective, of course – to an obligation with the Basel Accords, issued by the Basel Committee on Banking Supervision. These accords translate in a variety of recommendations regarding the Bank's own risk awareness.

In a way of following the Basel Accords, the insurance industry and its supervisors, represented by the European Insurance and Occupational Pensions Authority (EIOPA) in the European continent, are also spreading an "own risk self-assessment" (ORSA) culture through the Solvency II requirements. In this scenario, main objective of the Capital Management is to deliver a number – called onwards as risk capital – that represents the amount of money that the company has to set aside from its shareholders to use it as a cushion to protect the company from unexpected losses from the risks faced by the company.

The following example has the intention to clarify what risk capital is to a company: imagine that an insurance company sells life insurance to a hundred people and each policy pays $100 in case of death. If the company wants to be as conservative as one can be, the right side of its balance sheet – liabilities and shareholders' capital – will sum up to $10,000, which will cover the liability for the death of all policyholders. On the other hand, if the company wants to be as aggressive as one can be, it will have zero liabilities and zero capital, betting that no policyholder will die during the contract. Both scenarios are not possible, for regulatory reasons and for business reasons, once it might be difficult to convince shareholders to abide this kind of capital on the conservator scenario. Therefore, in the real world where we live in, somewhere in between these two scenarios, one has to study methods to quantify the uncertainty that arises with risky business.



The insurance industry supervisors, in some areas, demands that the insurance company discloses in its financial statements how much is its Minimum Capital Requirement (MCR) to operate and compare it with how much capital the company has. This gives the external user of the financial statement two different intel: how risky is the business of this insurance company and how solvent the company is. Usually the insurance regulator also delivers a called “Standard Formula” to calculated the MCR if a company does not want to send its own model to calculate it so the regulator can approve it or does not have a proper model. This standard formula usually works well on medium size companies with a diversified portfolio, but calculates a large MCR for small companies and a small one for large companies, compared as the one that should have been calculated using a proper method.

In this context, this project tries to contribute proposing a different model to calculate this “capital”, focused on the reserve/underwriting risk of a life insurance company, and, also, tries to investigate how different economic and biometric changes and developments may affect this number calculated.

This project is organized in four different sections, excluding this introductory section. The first one called “Model Development” is where the mathematical framework is explained and defended for the development of a formal model that will be built up further on the project. On the building process, the idea of a stochastic actuarial liability using the concept of nested simulations will be introduced, and, then, the formal model will be built.

The second section called “Simulated Results” will illustrate results on what will be called “Standard Scenario” with some specific and general assumptions and, then, four different stress scenarios will be created to understand how uncertainty on the financial market and developments on the mortality pattern affects this model.

On the third section called “Model In Practice and Comparison with the Brazilian regulator *standard formula*” will be demonstrated how the proposed model works on a fictional population and it will be compared the results of this model with the SUSEP’s (Brazilian insurance regulator) standard formula to calculate the risk capital.

Finally, on the final section called “Conclusion”, comments on the whole project will be made and further ideas for possible deeper research will be pointed out.

2. Model Development

2.1. Liability Modelling

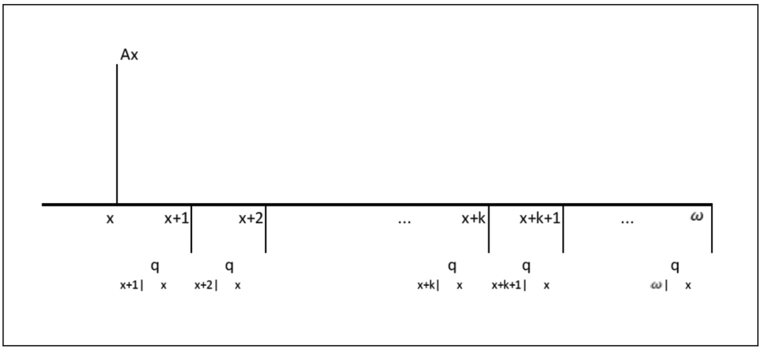
In order to accomplish the goals of this paper, a stochastic point of view must be taken while looking at what constitutes the actuarial liability, booked in the company's balance sheet, of a life insurance product. Along this project, as told before, only one type of life insurance product will be studied, unless stated otherwise: **a simple whole life insurance** payable at the end of the year of death and funded by a lump sum premium paid when the policy was issued.

The traditional and deterministic method to calculate the actuarial liability, widely known by practitioners and well defined by Jordan's contribution to the actuarial literature, Life Contigencies, is dependent on the actuary's judgement about two parameters: the mortality behaviour and the interest rate expectation. For the life insurance product that is been studied by this project, the actuarial liability is mathematically given by, per monetary unit assured:

$$A_x = \sum_{k=0}^{\omega-x} {}_{k|}q_x \times \left(\frac{1}{1+i}\right)^{k+1} \text{, } {}_{k|}q_x = {}_{k}p_x \times q_{x+k}$$

Where x is the age the policyholder, w is the final age of the life table, $k \mid q_x$ is the deferred for k years probability of death of a life of age x, defined above by a product of probabilities of surviving until the age $x+k$ and dying before reaching the next integer age, and i is the expected interest rate.

Some readers may prefer to understand this mathematical framework via a cashflow model, as the following diagram.

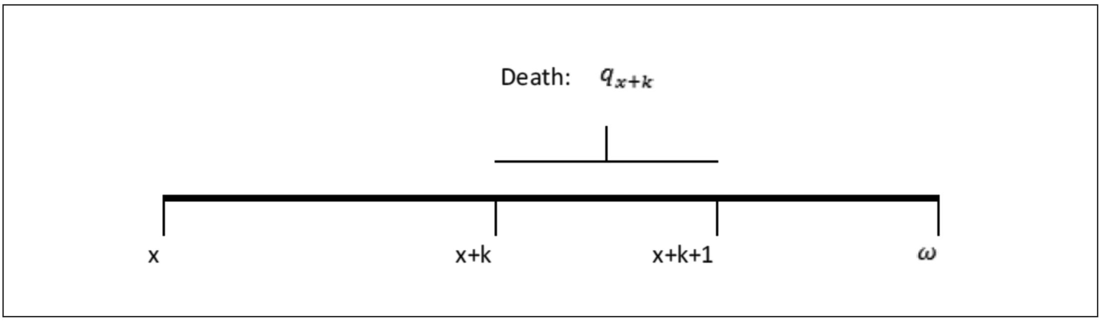


Via this equation, one can easily notice that both the mortality and the interest rate are random variables with its own statistical distributions, which means that A_x, the present value of a uncertain cashflow, is a random variable.

2.1.1. The mortality pattern as a random variable using a life table

A traditional life table provides a crucial information about the mortality pattern that is going to be used when modelling the stochastic actuarial liability: the probability of a life of a certain integer age dies before reaching the next integer age, also known as q_{x+k}. The following diagram illustrates the event:

Diagram 1 – the probability of a life aged $x + k$ dies at any point of time before its $(x + k + 1)$th birthday



With that, it will then be defined a random variable called K_x that represents the integer part of the future life-time of an individual at age x. This random variable can assume values from 0 to $(\omega - x)$. The probability mass function of this discrete random variable is mathematically given by:

$$P(K_x = k) = {}_{k|}q_x$$

And its cumulative distribution function is given by:

$$F(K_x) = P(K_x \leq n) = \sum_{k=0}^{n} {}_{k|}q_x$$

Therefore, to simulate a random sample for K_x is quite straightforward through the use of the inversion function of the cumulative distribution function:

$$F^{-1}\left(P(K_x \leq n)\right) = n$$

The Microsoft's Excel software will be used to all calculations henceforth. To create this sample, a vector with five hundreds random numbers drawn from a Uniform distribution with parameters 0 and 1 will be created using the "=RAND()" function. Then, the following function will be created to translate these random numbers into a random sample for K_x:

$$k_{x,z} = \begin{cases} 0, if\ 0 \le u_z \le (q_x) \\ 1, if\ (q_x) < u_z \le \left(\sum_{k=0}^{1} {}_{k|}q_x\right) \\ \vdots \\ n, if \left(\sum_{k=0}^{n-1} {}_{k|}q_x\right) < u_z \le \left(\sum_{k=0}^{n} {}_{k|}q_x\right) \\ \vdots \\ \omega - x, if \left(\sum_{k=0}^{\omega-x-1} {}_{k|}q_x\right) < u_z \le 1 \end{cases}, 1 \le z \le 500$$

In this part, it will only be used the life table "Annuity 2000 Basic – Male" (AT2000M), provided by the mortality library of the Society of Actuaries (SOA) under the identity number 885. The following table illustrates the life table's information for certain ages:

Table 1 – AT2000M life table's information for certain ages

Age	q_x	Age	q_x	Age	q_x
5	0.000324	20	0.000549	60	0.007170
10	0.000390	30	0.000784	80	0.051128
15	0.000470	40	0.001043	100	0.249741

In order to make sure that the simulation procedure is consistent, the age 40 was chosen and the procedure was launched to generate a random sample for K_{40} using the AT2000M mortality pattern. The quality of the simulation will be measured qualitatively by the behaviour of the locations measures (theoretical expectation and sample average) and the dispersion measures (theoretical standard deviation and sample standard deviation), both in years units:

Table 2 – K_{40} simulation adequacy

	Theoretical	Empirical	Diff. (%)
μ_{K40} and $\bar{x}$, respectively	21.449	21.528	-0.37
σ_{K40} and s, respectively	8.230	8.097	1.62

To continue to certify the procedure, seems appropriate to run it for a different age and compare the same measures. An age of 80 was chosen this time and the following table was produced:

Table 3 – K_{80} simulation adequacy

	Theoretical	Empirical	Diff. (%)
μ_{K80} and $\bar{x}$, respectively	4.0739	4.1320	-1.43
σ_{K80} and s, respectively	2.4118	2.4320	-0.84

It seems that the procedure's output is consistent, at least visually, by these measures and to carry out further tests seems inappropriate at this time.

2.1.2. The interest rate as a random variable in the liability modelling

The deterministic method to calculate an actuarial liability of a life insurance product usually uses the interest rate as either a flat rate through time – for instance, future cash flows will be discounted with a 4% interest rate per year – or as a term structure of interest, with interest rates varying along the time horizon. Both methods do not allow the interest rate to vary as a random variable in each period of time, hence are seen as deterministic approaches. Even for a term structure of interest rates, where each future rate is estimated, is still an expectation of what the number will be – except in cases where all investment bonds used to calculate the term structure were not priced via market data, but this case is rather unusual, where the interest rates are not an expectation, nor a random variable.

For the purposes of this project, the following model was chosen to allow the interest rate, which will be used to calculate the present value of the future cash flows, to be a random variable:

$$ln\left(S_{K_x+1}\right) \sim ln\left((1+i)^{(K_x+1)}\right) \sim N\left((K_x+1)\times\mu, (K_x+1)\times\sigma^2\right)$$

This model implies that the accumulation – the inverse of the "discount" procedure – behaves as a Log Normal Distribution with parameters $\mu^* = (K_x + 1) \times \mu$ and $\sigma^{2*} = (K_x + 1) \times \sigma^2$.

When implementing this calculation, one has to carefully estimate the mean and the standard deviation so the output of the model will be consistent with one's investment portfolio. For instance, on portfolios in which there are relatively more allocation on equities, it is expected that its standard deviation is higher than a portfolio with all allocation on bonds. From now onwards, the following assumption stands: the expectation and the standard deviation of i, as a random variable, is 4% and 0.5%, respectively. Which means that μ and σ, the expectation and the standard deviation of $\{ln(1 + i) – 1\}$, is 3.92% and 0.48%, in rounded terms, respectively.

To simulate a random sample for $D_{K_x+1} = \frac{1}{S_{K_x+1}}$, the procedure is similar to the one presented before, but Microsoft Excel has a built-in function to invert the cumulative distribution function of a Log Normal distribution. The function used was "=1/(LOGNORM.INV(RAND(), μ^*, σ^*))", once what has been modelled is the discount factor and not the accumulation factor. A hundred numbers were generated for each $k_{x,z}$. Which means that fifty thousand (500 times 100) numbers were generated in total.

2.1.3. The stochastic actuarial liability

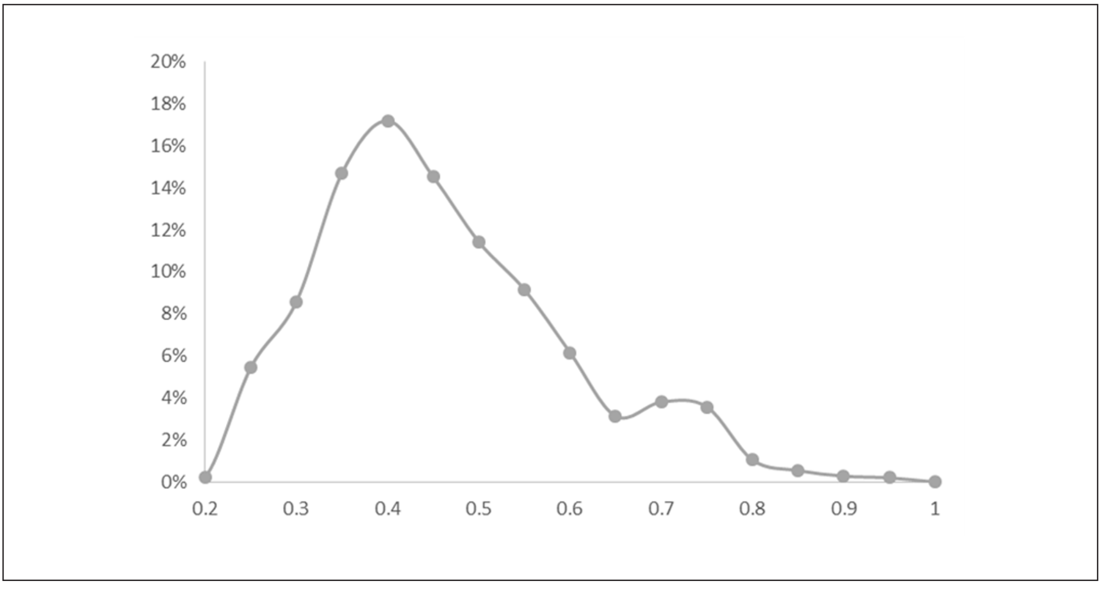
The actuarial liability for this product, as mathematically enunciated before, depends basically of two random variables: the mortality pattern and the interest rate. In the previous sections, it was defined how this project will allow the mortality and the interest rate to vary according to an specific distribution through simulations, or nested simulations – a procedure to simulate within simulations – as defined in Bauer (2015). Visually, the following matrix was created:

$$\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & . & . & . & 99 & 100 \\ \hline
1 & & & & & & & & \\
2 & & & & & & & & \\
3 & & & & & & & & \\
. & & & & A_x\left(K_{x,z}, D_{(K_{x,z},j)}\right) & & & & \\
. & & & & & & & & \\
. & & & & & & & & \\
499 & & & & & & & & \\
500 & & & & & & & &
\end{array}$$

Where $1 \le z \le 500$ and $1 \le j \le 100$. This matrix can be interpreted as a random sample for A_x, previously defined.

In order to confirm, it will, once again, be shown a measure for two specific ages. The expectation of A_x is easy to calculate on both the theoretical and the simulated scenario. For A_{40} using the AT2000M mortality pattern and $E(i) = 4\%$, the present value of this future cash flow is 0.4369 in monetary units per unit assured. The following histogram was created with the $A_{40\left(K_{40,z},D_{\left(K_{40,z},j\right)}\right)}$ sample:

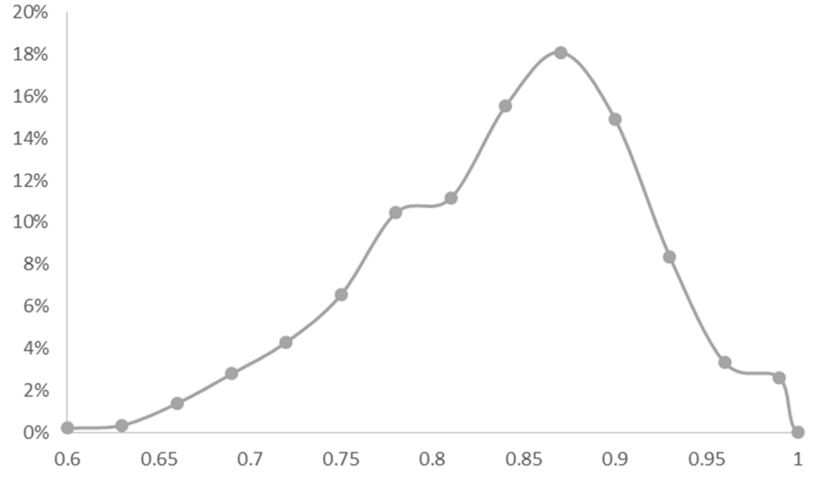
Graph 1 – Histogram of $A_{40\left(K_{40,z},D_{\left(K_{40,z},j\right)}\right)}$



And the average of this sample is 0.4362 in per monetary unit assured, which is 0.13% of difference of the theoretical expectation.

For A_{80} using the AT2000M mortality pattern and $E(i) = 4\%$, the present value of this future cash flow is 0.8232 in monetary units per unit assured. The following histogram was created with the $A_{80}\left(K_{80,z}, D_{(K_{80,z},j)}\right)$ sample:

Graph 2 – Histogram of $A_{80}\left(K_{80,z}, D_{(K_{80,z},j)}\right)$



And the average of this sample is 0.8263 in per monetary unit assured, which is -0.38% of difference of the theoretical expectation.

2.2. Risk Capital Modelling

One of the goals of this project is to come up with a useful and different model to calculate the risk capital that a life insurance company must keep (Capital), as the definitions of the QIS 5 Technical Specifications for Solvency II, issued by the European Commission (2010), and then build it up to achieve its other goals.

Differently from the general insurance practice, life insurance and other capitalized products usually calculates the Capital based on the definition of Available Capital (AC), as in Bauer (2015), or Net Asset Value (NAV), as in Le Courtois (2014), which is similar concepts and both also relies on nested simulations to computationally achieve its objectives in a much more complex definition. The AC is defined as the difference between the present value of assets and the present value of liabilities at any given time.

Using Bauer's work, the Capital can be defined as:

$Capital \coloneqq argmin_x \left\{ P\left(AC_0 - \frac{AC_1}{1-s(0,1)} > x \right) \leq 1 - 99.5\% \right\}$, where $s(0,1)$ is the discount rate of a one-year zero-coupon bond.

For non-capitalized products – general insurance – the equation can be similar:

$Capital \coloneqq argmin_x \{P(Y - E(Y) > x) \leq 1 - 99.5\%\}$, where Y is the random variable associated with the value of the reserve.

Which is also the definition of the difference between value-at-risk with 99.5% of confidence, $VaR_{99.5\%}(Y)$, of the Loss Distribution given by Y and the expectation of this distribution.

It is difficult to point out just one definitive measure to calculate the Capital. Sometimes the $VaR_{99.5\%}(Y)$ is not enough and the expected shortfall – or Tail-Value-at-Risk – concept must be incorporated. Hence, in this context of different measures, this project tries to make a difference.

2.2.1. The proposed risk capital model

One of its goals to propose a model to calculate the reserve risk capital – an amount of money that a company must hold to cover adverse development of its losses – of a capitalized insurance product.

2.2.1.1. Model introduction

For the specific product that is been studied by this project, the actuarial liability is given by A_x, or $E(A_x)$, as shown before. As also shown in the previous steps, it was developed a consistent method to simulate random values for A_x. In order to fully understand how to build up to the model, the following need to be defined:

- $A = VaR_p(\Delta) = argmin_x \{P(\Delta \geq x) = 1 - p\}$, where Δ is a general loss distribution;
- $B = TVaRp(\Delta) = E\left(\Delta | \Delta \geq VaR_p(\Delta)\right)$, known as Tail-Value-at-Risk.

Using Microsoft Excel's built-in functions, A is calculated by the formula "=PERCENTILE.INC(A_x sample matrix, p)" and B by the formula "=AVERAGEIFS(A_x sample matrix, A_x sample matrix,">="& A)".

As said before, for general insurance purposes, the Capital is simply the difference between either A or B and the expectation of the loss distribution, once usually risks written by general insurance companies has a year of policy cover.

For a whole-life insurance product, however, the policyholder has his/hers remaining life covered, which means that the insurance company will be setting aside more funds than should have if its risk capital model was the same as for general insurance products. So the fundamental question now is how to allocate all this capital that is due to the whole remaining life of the policyholder for the next year.

2.2.1.2. One-year allocation for a remaining life due capital

To develop a consistent solution to the said problem, it is natural to think about what impacts the value of the actuarial liability in this: the age of the policyholder. Everything that was done until now was leaving x as a given value, but now it made sense to study more about this number. A natural manner to study the relationship about the age x of the policyholder and the variables of the model is by its effects. For instance, the youngest the policyholder is, the lowest its allocated actuarial liability per unit assured will be, once A_x is clearly an increasing function regarding the age. Another important relationship is that the oldest the policyholder is, the lowest its associated standard deviation of the future lifetime, as pointed out in "2.1.1." section. Finally, it is rather easy to think that the oldest the policyholder is, less time the company will have to cover deviations in its reserve, so more of the capital calculated in the previous step will be allocated to this year.

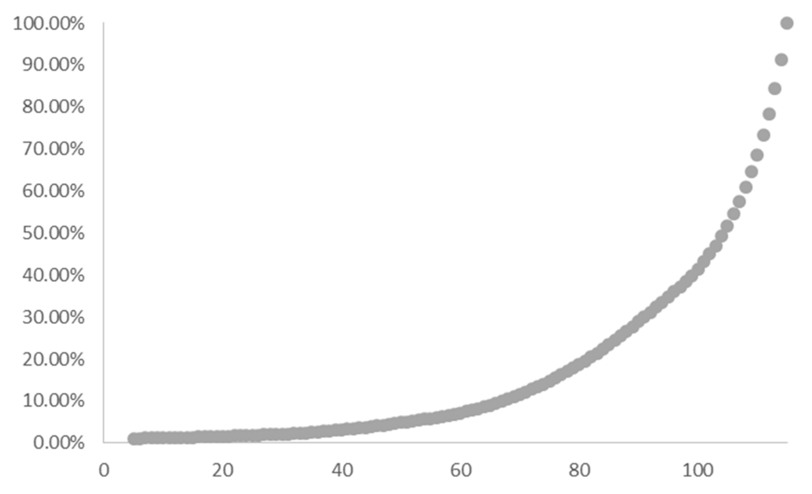
Therefore, if the capital calculated will be allocated to the following year through the use of a factor, this factor has to behave as an increasing function with the age of the policyholder. And more, has to start at the zero neighbourhood and to finish at 1, where all the Capital calculated has to be allocated that year.

The factor chosen was:

$$Factor_x = \frac{A_x}{\ddot{a}_x} \times (1 + i)$$

The following graph shows the behaviour of the $Factor_x$ regarding the age:

Graph 3 – Factor behaviour with age



As shown, this factor complies with the conditions that were expected.

2.2.1.3. Formal model

Based on the previous steps, the proposed model has this simple formula:

$$Capital(p,x,z,j,\mu_i,\sigma_i) = Factor_x \times \left\{TVaR_p\left(A_{x\left(K_{x,z},D_{(K_{x,z},j)}\right)}\right) - E\left(A_{x\left(K_{x,z},D_{(K_{x,z},j)}\right)}\right)\right\}$$

Which means that the Capital is a function of the significance level used to calculate the Tail-Value-at-Risk, the age of the policyholder, the number of iterations to simulate the future lifetime of policyholder, the number of iterations to simulate the economic scenarios and finally the expected value, the standard deviation of the interest rate and, implicitly, the expected mortality pattern.

In order to have a visually more interesting discussion, that do not depend on the individual sum assured, it will be analysed the increase in the booked reserve due to the capital calculation:

$$InCRes(p,x,z,j,\mu_i,\sigma_i) = \frac{\left\{Capital(p,x,z,j,\mu_i,\sigma_i) + E\left(A_{x\left(K_{x,z},D_{(K_{x,z},j)}\right)}\right)\right\}}{E\left(A_{x\left(K_{x,z},D_{(K_{x,z},j)}\right)}\right)} - 1$$

Once all parameters are comfortably estimated, the final product of this model will be a table, such as a life table, of percentages by age. Then, to calculate the Capital one has to simply multiply this number, expressed as a percentage, by the actuarial liability individually calculated.

2.2.1.4. Confidence intervals for the Capital

At each run of the model, with the same parameters, a different value for $InCRes\ (p,\ x,\ z,\ j,\ \mu_i,\ \sigma_i\ )$ will be generated, once it is dependent on simulated values. In order to understand this variability and with that, deliver confidence intervals for the Capital calculated, a sample for $InCRes(p,x,z,j,\mu_i,\sigma_i)$ was created via two hundred simulations and the average, lower bound and upper bond was calculated, with 0.5% of significance.

- $R_1 = argmax_x\left\{P\left(\underbrace{InCRes(p,x,z,j,\mu_i,\sigma_i)} \leq x\right) = 0.5\%\right\}$
- $R_2 = E\left(\underbrace{InCRes(p,x,z,j,\mu_i,\sigma_i)}\right)$
- $R_3 = argmin_x\left\{P\left(\underbrace{InCRes(p,x,z,j,\mu_i,\sigma_i)} \geq x\right) = 99.5\%\right\}$

Where $\underbrace{X}$ represents a sample of the random variable X. The empirical results R_1, R_2 and R_3 will be presented in the following section.

3. Simulated Results

Using the mortality pattern from the AT2000M life table, five hundred iterations for the future lifetime, a hundred iterations for the interest rate scenarios with mean and standard deviation for the interest rate of 4% and 0.5%, respectively, the following results were produced, by age and significance level to calculate the Tail-Value-at-Risk.

Table 3 – Standard results for a set of x's and p's given the other parameters

$x = 30$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.086%	2.384%	2.785%
R_2	2.457%	2.837%	3.395%
R_3	2.821%	3.203%	3.818%
$x = 50$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.532%	2.781%	3.101%
R_2	2.846%	3.141%	3.536%
R_3	3.130%	3.458%	3.860%
$x = 70$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.799%	2.978%	3.272%
R_2	3.054%	3.286%	3.486%
R_3	3.316%	3.579%	3.698%
$x = 90$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.700%	2.763%	2.868%
R_2	2.878%	2.944%	3.059%
R_3	3.065%	3.140%	3.262%

The first important trend to notice is an increase from left to right on the values of table. Which is expected once lower significance levels (higher values for p) translates in higher values for the Tail-Value-at-Risk, and, therefore, higher values of Capital. Another important trend that is worth to highlight is how the results for R_1, R_2 and R_3 are getting closer to each other as the age is higher (up to down), which was also expected once, as showed in the "2.1.1." section, the standard deviation of the simulated future lifetime decreases as the age increases.

These visual trends are important to measure the consistency of what was simulated and to notice if any of the calculations are delivering something strange.

3.1. First stress-test on the Standard Results

For this first stress-test, only one parameter was changed to understand how it affects the estimation process: the standard deviation for the interest rate – σ_i – 1.0% instead of 0.5% as before. The other parameters were all kept the same and the following results were produced:

Table 4 – Standard results for a set of x's and p's given the other parameters with σ_i – 1.0% instead of 0.5% as before

$x = 30$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.110%	2.403%	2.831%
R_2	2.463%	2.844%	3.407%
R_3	2.823%	3.207%	3.823%
$x = 50$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.550%	2.795%	3.141%
R_2	2.855%	3.152%	3.554%
R_3	3.136%	3.465%	3.867%
$x = 70$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.825%	3.031%	3.341%
R_2	3.077%	3.315%	3.566%
R_3	3.332%	3.575%	3.771%
$x = 90$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.857%	2.981%	3.202%
R_2	3.018%	3.152%	3.386%
R_3	3.189%	3.331%	3.575%

As a general rule, all values were higher than the ones presented before, once the increase on the uncertainty about the interest rates made the sample of A_x to show a higher deviation about its average and increasing, then, the Tail-Value-at-Risk.

However, in a more careful analysis, the increase was higher on higher value of x than it was for smaller values of x. This is an effect of the $Factor_x$, once more of the increase on the deviation has to be incorporated in the Capital at older ages.

3.2. Second stress-test on the Standard Results

For this second stress-test, it is only natural to try to investigate, although the answer may be expected, what will happen with the model if the mean of the interest rate were changed, instead of the standard deviation. The other parameters were all kept the same as in the standard results, expect for μ_i that now assumes 8% instead of 4%, and the following results were produced:

Table 5 – Standard results for a set of x's and p's given the other parameters with $\mu_i = 8.0\%$ instead of 4.0% as before

$x = 30$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	3.28444%	3.91113%	4.89271%
R_2	3.98695%	4.86373%	6.28115%
R_3	4.69820%	5.66516%	7.39545%
$x = 50$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	4.74078%	5.25883%	5.89098%
R_2	5.18903%	5.85795%	6.77881%
R_3	6.03784%	6.60109%	7.57270%
$x = 70$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	5.21800%	5.71487%	6.15601%
R_2	5.82213%	6.31681%	6.72981%
R_3	6.36397%	6.91778%	7.18524%
$x = 90$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	5.06685%	5.12974%	5.23686%
R_2	5.48732%	5.55539%	5.67370%
R_3	5.92105%	5.99346%	6.11714%

These numbers shows a pattern similar to those ones first produced regarding trends, but with a significant increase in its value. This effect is explained by what this number is: the increment in the reserve due to risk capital. With a higher expected value for the mean of the interest rate, it is expected that the value of the reserve decreases in number. As most of the variability of the Capital is due to the variability in the future lifetime, the amount of the capital needed had not changed much. Therefore, to keep a similar amount of Capital with less reserve, these increases – numbers shown in the table – had to be higher than before.

3.3 Third stress-test on the Standard Results

In order to understand how the mortality pattern, translated in this model by q_x's of a life table, affects the need of Capital to cover the risk of adverse developments on the mortality, the natural solution is to change this parameter and investigate what will happen with the model's output.

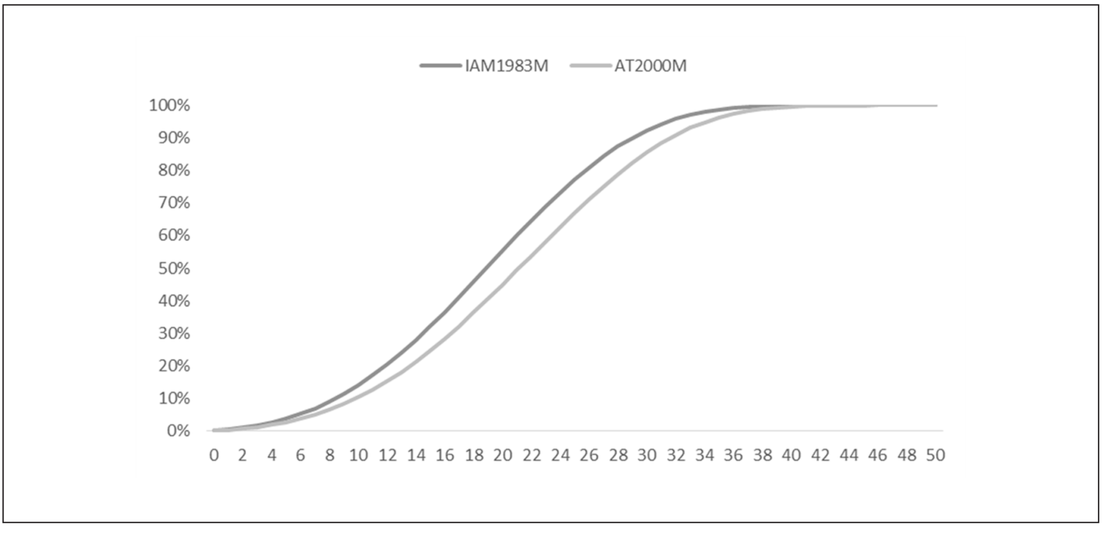
For this third and last stress-test, the mortality pattern will be changed twice. In the first change, the AT2000M q_x's will be replaced by the q_x's of the table "1983 IAM Basic – Male" (IAM1983M) provided by the mortality library of the Society of Actuaries (SOA) under the identity number 824. The q_x's for some ages of this table are presented below:

Table 6 – IAM1983M life table's information for certain ages

Age	q_x	Age	q_x	Age	q_x
5	0.000419	20	0.000559	60	0.009266
10	0.000424	30	0.000850	80	0.063132
15	0.000483	40	0.001476	100	0.300716

To make this change visually understood by the reader, the cumulative distribution function (CDF) of K_x, the integer part of the future lifetime of an individual aged x, were plotted for both life tables and for $x = 40$. The results are shown below, note that the graph it is only showing the value of the CDF until $K_{40} = 50$, once for higher values it is already very close to one:

Graph 4 – K_{40} cumulative distribution function for both AT2000M (standard scenario) and IAM1983M (stress scenario)



By this chart, it is easy to notice that IAM1983M life table illustrates higher q_x's, as one may have expected, once the common sense says that life tables built by an older mortality experience tend to produce higher q_x's. However, is this a reason to need more Capital?

The following table was produced by replacing the q_x's of the AT2000M life table for the ones produced by the IAM1983M life table. Every other parameter was kept the same as in the first result shown.

Table 7 – Standard results for a set of x's and p's given the other parameters and mortality pattern from the IAM1983M life table

$x = 30$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.15454%	2.43950%	2.86076%
R_2	2.49184%	2.85471%	3.39637%
R_3	2.84874%	3.19996%	3.79370%
$x = 50$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.68971%	2.95094%	3.23827%
R_2	2.91950%	3.19833%	3.55502%
R_3	3.31018%	3.53183%	3.86546%
$x = 70$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.77652%	2.97240%	3.18903%
R_2	3.05241%	3.29091%	3.45729%
R_3	3.29813%	3.53346%	3.64165%
$x = 90$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.63695%	2.70644%	2.83088%
R_2	2.81183%	2.87988%	3.00142%
R_3	3.00755%	3.08964%	3.22148%

As before, all the expected trends within the table are easy to notice. However, it is not possible to notice any abnormal trend in the change between both results. In a closer look and fixing $p = 99.5\%$, for instance, one can notice that for the two youngest ages, the results generally decreased, and for older ages the opposite happened.

Bearing in mind that, due to the heavier mortality pattern, the average of the actuarial reserve distribution will certainly increase, an increase in the results shown means an even higher increase in the need of capital. Also due to that, a decrease in the results shown for the youngest ages do not mean necessarily a decrease in the need of capital. However, for younger ages, the one-year correction factor, is also responsible to produce lower capital need, as the policyholder has a higher average future lifetime.

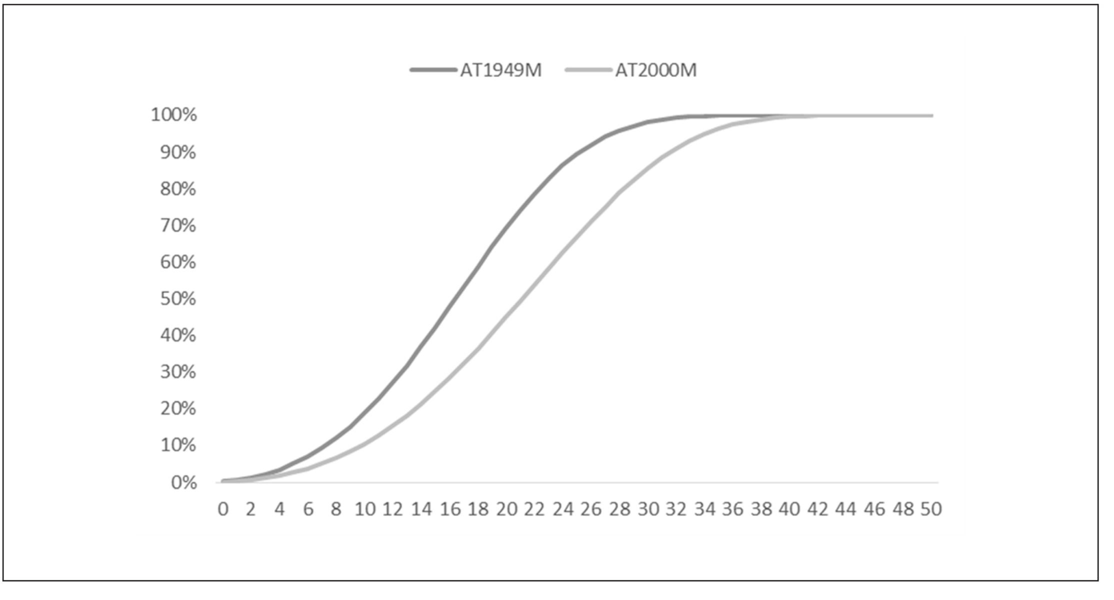
However, in order to be even more certain about the answer for the posed question, a second change in the mortality pattern will be done. From now on, the AT2000M q_x's will be replaced by the q_x's of the table "a-1949 with Extension– Male" (AT1949M) provided by the mortality library of the Society of Actuaries (SOA) under the identity number 808. The q_x's for some ages of this table are presented below:

Table 8 – AT1949M life table's information for certain ages

Age	q_x	Age	q_x	Age	q_x
5	0.000566	20	0.000624	60	0.015662
10	0.000483	30	0.001004	80	0.085503
15	0.000537	40	0.002025	100	0.463415

Similar to earlier on this section, the cumulative distribution function (CDF) of K_x, the integer part of the future lifetime of an individual aged x, were plotted for both life tables and for $x = 40$. The results are shown below, note that the graph it is only showing the value of the CDF until $K_{40} = 50$, once for higher values it is already very close to one:

Graph 5 – K_{40} cumulative distribution function for both AT2000M (standard scenario) and AT1949M (stress scenario)



As one can easily notice, the mortality experience translated by the AT1949M produces even higher q_x's than the IAM1983M life table, measured by the distance between the two curves and the standard scenario.

The following table was produced by replacing the q_x's of the AT2000M life table for the ones produced by the AT1949M life table. Every other parameter was kept the same as in the first result shown.

Table 9 – Standard results for a set of x's and p's given the other parameters and mortality pattern from the AT1949M life table

$x = 30$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.25959%	2.56390%	2.93140%
R_2	2.53216%	2.89431%	3.39475%
R_3	2.78796%	3.20283%	3.79305%
$x = 50$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.66069%	2.94381%	3.22450%
R_2	2.95243%	3.22262%	3.55605%
R_3	3.22795%	3.53763%	3.84396%
$x = 70$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.84952%	3.09560%	3.18094%
R_2	3.07911%	3.27794%	3.38793%
R_3	3.37186%	3.48725%	3.58247%
$x = 90$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2.44486%	2.51617%	2.64451%
R_2	2.63440%	2.70668%	2.84016%
R_3	2.87106%	2.94900%	3.09180%

The results are consistently very similar to the ones originally produced, except, however, by a significant decrease for $x = 90$, explained by the lower volatility of both K_{90}'s the higher average for the K_{90} of the standard scenario.

Therefore, it seems fair and reasonable to answer that yes, a heavier mortality pattern will generate a higher capital, even though the capital is used to cover adverse development of the risks only for the next year.

3.4. Fourth stress-test on the Standard Results

As an exercise to see how the choice of using the Tail-Value at Risk in the model affect its results, a results table was made using the definition of Value at Risk to calculate the risk capital. The results found are shown below:

Table 10 – Standard results applying the concept of Value at Risk instead of Tail-Value at Risk

$x = 30$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	1,48200%	1,96125%	2,57889%
R_2	1,83781%	2,35885%	3,18385%
R_3	2,15822%	2,82722%	3,61204%
$x = 50$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2,11962%	2,47933%	3,03720%
R_2	2,37180%	2,79981%	3,39248%
R_3	2,78136%	3,16548%	3,82924%
$x = 70$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2,33554%	2,68391%	3,15050%
R_2	2,66640%	2,96669%	3,43902%
R_3	2,93557%	3,44612%	3,63589%
$x = 90$	$p = 95.0\%$	$p = 97.5\%$	$p = 99.5\%$
R_1	2,58898%	2,67104%	2,80017%
R_2	2,77408%	2,86523%	3,00513%
R_3	2,96470%	3,05949%	3,21500%

As expected the increments are significant lower than the ones presented before. This shows how conservative is the choice of using the Tail-Value at Risk in the model.

4. Model in Practice and Comparison with the Brazilian Regulator *Standard Formula*

The objective of this section is to implement this model on a fictional population and then compare it with the results of a "Standard Formula" calculation. For the purposes of this paper, the following information about the population is given, bearing in mind that the actuarial assumptions come from the standard scenario presented before:

Table 11 – Fictional population information

Classification	# number of policyholders	Individual Sum Assured	Age	Exposure (in $M)	Actuarial Liability
1	1,000	$150,000	30	$150,000	$51,331
2	650	$100,000	50	$65,000	$35,493
3	200	$70,000	70	$14,000	$10,409
4	150	$50,000	90	$7,500	$6,587
Total	**2,000**			**$236,500**	**$103,819**

In this scenario, the proposed model's output, in terms of Capital, is, in $M, as follows:

Table 12 – Proposed Risk Capital, in an increasing confidence degree

Classification	$R_{1,p=95.0\%}$	$R_{2,p=97.5\%}$	$R_{3,p=99.5\%}$
1	1,078	1,458	1,958
2	938	1,118	1,372
3	287	341	383
4	177	194	216
Total	**2,480**	**3,112**	**3,930**

In this scenario, the proposed model's output, using the concept of Value at Risk, instead of the Tail-Value at Risk, in terms of Capital, is, in $M, as follows:

Table 13 – Proposed Risk Capital (2), in an increasing confidence degree

Classification	$R_{1,p=95.0\%}$	$R_{2,p=97.5\%}$	$R_{3,p=99.5\%}$
1	761	1,211	1,854
2	752	994	1,359
3	243	309	378
4	171	189	212
Total	**1,927**	**2,702**	**3,803**

Therefore, for this fictional insurance company, the Management has to define whether or not it wants to be aggressive or conservative in terms of deviations from the expected assumptions for the next year and translate it into an amount to set aside as Risk Capital.

As told in the first section, if a company does not have a proper model to calculate the Minimum Capital Requirement, it can use a standard formula to generate it for disclosure of solvency requirements. The Brazilian insurance regulator, called *Superintendencia de Seguros Privados* (SUSEP), demands since 2010 that every insurance company has to disclose in its financial statements the minimum risk capital required for its operation. An enormous actuarial work has been done by SUSEP to achieve a Standard Formulae similar to the EIOPA's one and apply it to the Brazilian insurance framework.

The SUSEP's Standard Formula to calculate risk capital segregates the company's operation in Property/Casualty (called *Danos*) and Life/Pensions (called *Vida Individual e Previdência*) and then requires the company to populate information such as the lump sum payments made over the last 12 months, or premium received by region. For the specific product, classified by SUSEP as a risk of death coverage for capitalized products paid with a lump sum payment and it is calculated simply by multiplying the actuarial liability to a factor of 0.25%, 1.70% or 3.21% depending on the interest rate (i) used to calculate the actuarial liability. The first factor is for "i" greater than zero and less than three percent, the second one is for "i" between 3% and 6% and the last one is for "i" greater than six percent.

With that, the following table can be presented:

Table 12 – Risk Capital calculation and comparison

Classification	$R_{1,p=95.0\%}$	$R_{2,p=97.5\%}$	$R_{3,p=99.5\%}$	SUSEP's Calculation
1	1,078	1,458	1,958	767
2	938	1,118	1,372	668
3	287	341	383	204
4	177	194	216	126
Total	**2,480**	**3,112**	**3,930**	**1,765**

The number calculated by SUSEP's methodology is lower than the output of the model, but is similar to the one calculated using the Value-at-Risk concept.

5. Conclusion

As presented in the first section, the main objective of this project was to propose a different method to calculate risk capital of a life insurance company focused only on the reserve/underwriting risk itself, not on market risk, credit risk, or operational risk. And, once this method was defined, to investigate how changes and/or development on certain assumptions affect the model and its outputs.

An important achievement of this project is, in the Brazilian scenario at least where it was tested, is the comparability of the model proposed, in terms of output, with the Standard Formula proposed by the insurance supervisor. The results are similar – especially if VaR is used instead of TVaR – and a company may use it do adequate the risk capital necessity to their own reality of interest rates and biometric characteristics.

The risk capital model framework illustrated in this paper has a clearly advantage over other models that is its simplicity on the calculation itself, once the number can be individually calculated simply by multiplying the calculated actuarial liability by the factor. If an insurance company wants, it is possible to produce a table of factors by age and just manage the assumptions and the estimates. But also, one has to bear in mind that the product that the whole project is based on is a very simple product.

The simplicity of the product in this case is where the first improvement for this project resides: to understand how the model will behave on other types of products. For instance, if it was a term life assurance product, or a with-profit assurance product – or even more complex products with other guarantees.

Another question that this research does not answer is whether or not there are correlations between products of a diversified portfolio and estimate the benefits of this kind of diversification regarding capital management.

Finally, a very important further research that can be done is to compare this kind of approach with already existing models.

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